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Simulation of Resonant Modes of Rectangular DR in MIC Environment Using MPIE-MoM with Combined Entire-Domain and Sub-Domain Basis Functions

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Abstract **— An efficient method-of-moments volume integral formulation using combined entire-domain and sub-domain basis functions is proposed for simulating the resonant modes of a rectangular DR in MIC environment. The formulation is based on the Mixed-Potential Integral Equation using Michalski's Formulation-C Green's functions. The spatial-domain Green's functions are calculated by using the Complex Image Method. Different from the simple sinusoidal entire-domain basis functions used by previous methods, an approximate solution for the resonant modes of rectangular DR in MIC environment based on Marcatili's method is used as the entire-domain basis function, while a set of tetrahedral basis functions are used as sub-domain basis functions. Since the main profile of the resonant mode can be represented well by the entire-domain basis function, only a small number of sub-domain basis functions are needed for further refinement of the representation. Compared with the method-of-moments formulations using only sub-domain or entire-domain basis functions, this method is much faster for the same accuracy. Numerical results are given and compared with other methods.**

I. INTRODUCTION

Recently dielectric resonator antennas (DRA's) have been extensively used in microwave integrated circuits (MIC's) due to their low loss, small size and mechanical simplicity [1-2]. Rectangular DRA is favorable to cylindrical and hemispherical DRA because it is easier to fabricate and has more degree of freedom for controlling the resonant frequency and quality factor [2], but it is more difficult to simulate than the latter two since the latter two are bodies of revolution (BOR) and have efficient surface integral methods. The geometry of a rectangular DR in MIC environment is shown in Fig.1.

Method of moments (MoM) is a popular method for simulation of DR [3, 4]. Either tetrahedral sub-domain basis function [5] or sinusoidal entire-domain basis function [3, 4] can be used for MoM. The advantage of tetrahedral sub-domain basis function is its capability to conform to arbitrary shape. However, generally the size of the tetrahedron should be smaller than tenth of the wavelength in the dielectric, which results in large number of unknowns. Using sinusoidal entire-domain basis function results in fewer unknowns, but the high-order sinusoidal basis functions take long time to integrate while contribute little to the field, therefore the efficiency decreases as the order of the basis function increases.

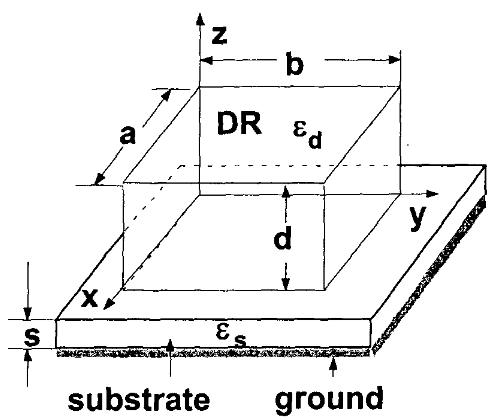

Fig.1. A rectangular DR in MIC environment.

To overcome the disadvantages of using sub-domain or entire-domain basis function, we propose a MoM formulation which uses combined entire-domain and sub-domain basis functions. Instead of using sinusoidal entire-domain basis functions, we use analytical approximate solutions [2] of the resonant modes of the DR as entire-domain basis functions. It takes little computation time to obtain the approximate solution since only several transcendental equations need to be solved. Although the resonant frequency and quality factor obtained from the approximate solution are not very accurate, the field distribution obtained from the approximate solution serves as a good entire-domain basis function which is close to the exact field. Therefore, combined with tetrahedral sub-domain basis functions, the field can be represented

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efficiently.

In our formulation the mixed-potential integral equation (MPIE) [6] is used in combination with Michalski's Formulation-C Green's functions [6] for multilayered medium. Discrete complex images method (DCIM) [7] is used for the calculation of spatial-domain Green's functions.

The organization of the summary is as follows: Section I briefly introduces the background and our contribution. Section II derives the formulation of the method. Section III gives some numerical results. Conclusions are presented in Section IV.

II. MPIE-MoM Formulation with Combined Sub-Domain and Entire-Domain Basis Functions

Consider a rectangular DR in MIC environment as shown in Fig.1. The dimension of the DR in x, y and z direction is a, b and d respectively, and it is assumed that $a \ge b \ge d$. The thickness of the substrate is s. The permittivity of the DR and the substrate is ε_d and ε_s, respectively.

Assume that the total field is $\vec{E}^{tot}$. According to the volume equivalence theorem [8], the DR can be replaced by equivalent volume current density (assuming $e^{j\omega t}$ time dependency)

$$\vec{J}_v(\vec{r}) = j\omega[\varepsilon_d(\vec{r}) - \varepsilon_0]\vec{E}^{tot}(\vec{r}) \quad (1)$$

where $\varepsilon_d(\vec{r})$ is the permittivity of the DR.

The electric field caused by $\vec{J}_v(\vec{r})$ is

$$\begin{aligned}\vec{E}^v(\vec{r}) &= -j\omega\vec{A}(\vec{r}) - \nabla\Phi(\vec{r}) \\ &= -j\omega\int_V \overline{\overline{G}}^A(\vec{r}|\vec{r}')\cdot\vec{J}_v(\vec{r}')dv' \\ &\quad -\nabla\int_V G^\Phi(\vec{r}|\vec{r}')q_v(\vec{r}')dv',\end{aligned} \quad (2)$$

where $\overline{\overline{G}}^A(\vec{r}|\vec{r}')$ and $G^\Phi(\vec{r}|\vec{r}')$ are the Formulation-C Green's functions for multilayered medium [6],

$$q_v(\vec{r}) = -\frac{1}{j\omega}\nabla'\cdot\vec{J}_v(\vec{r}). \quad (3)$$

In general cases, the total field is

$$\vec{E}^{tot}(\vec{r}) = \vec{E}^i(\vec{r}) + \vec{E}^v(\vec{r}). \quad (4)$$

where $\vec{E}^i(\vec{r})$ is the incident field. However, since we are interested in the resonant modes of the DR, therefore $\vec{E}^i(\vec{r}) = 0$. (4) is the integral equation to be solved.

To solve the integral equation through MoM, first we expand the electric flux $\vec{D}(\vec{r})$ with a set of vector basis functions $\{\vec{f}_n(\vec{r}) \mid n = 1,2,\cdots,N\}$. To simplify the representation, here we use a general form for both the entire-domain and sub-domain basis functions.

Assume the n-th basis function $\vec{f}_n(\vec{r})$ is defined on a set of polyhedrons $U_n = \{V_{np} \mid p = 1,2,\cdots,P_n\}$, and the surface of V_{np} is denoted as S_{np}. $\vec{f}_n(\vec{r})$ has continuous first-order derivatives in each V_{np}, but is not necessarily continuous on the interface between different V_{np}. $\varepsilon_d(\vec{r})$ is constant in each V_{np}, but may be different in different V_{np}.

The equivalent volume current due to the basis function $\vec{f}_n(\vec{r})$ is

$$\vec{J}_n^v(\vec{r}) = j\omega\kappa(\vec{r})\vec{f}_n(\vec{r}), \vec{r} \in U_n \quad (5)$$

where

$$\kappa(\vec{r}) = \frac{\varepsilon_d(\vec{r}) - \varepsilon_0}{\varepsilon_0} \quad (6)$$

After taking the Galerkin's testing procedure, the integral equation is converted into a matrix equation

$$[Z_{mn}][I_n] = 0 \quad (7)$$

where

$$\begin{aligned} Z_{mn} = \sum_{p=1}^{P_n}\sum_{q=1}^{P_m} \Big[& -j\omega\int_{V_{mq}}\int_{V_{np}} G^A(\vec{r}|\vec{r}')\vec{J}_n^v(\vec{r}')\cdot\vec{f}_m(\vec{r})dv'dv \\ & -\int_{S_{mq}}\int_{V_{np}} G^\Phi(\vec{r}|\vec{r}')\rho_n^v(\vec{r}')\hat{n}_{mq}\cdot\vec{f}_m(\vec{r})dv'ds \\ & +\int_{V_{mq}}\int_{V_{np}} G^\Phi(\vec{r}|\vec{r}')\rho_n^v(\vec{r}')\nabla\cdot\vec{f}_m(\vec{r})dv'dv \\ & -\int_{S_{mq}}\int_{S_{np}} G^\Phi(\vec{r}|\vec{r}')\rho_{np}^s(\vec{r}')\hat{n}_{mq}\cdot\vec{f}_m(\vec{r})ds'ds \\ & +\int_{V_{mq}}\int_{S_{np}} G^\Phi(\vec{r}|\vec{r}')\rho_{np}^s(\vec{r}')\nabla\cdot\vec{f}_m(\vec{r})ds'dv \\ & -\frac{1}{j\omega}\int_{V_{mq}\cap V_{np}} \vec{T}_m^v(\vec{r})\cdot\frac{\vec{J}_n^v(\vec{r})}{\varepsilon_d(\vec{r})-\varepsilon_s(\vec{r})}dv \Big] \end{aligned} \quad (8)$$

where $\hat{n}_{np}$ is the outward-pointed normal unit vector of V_{np}.

(8) is a general formulation for any basis functions which has continuous first-order derivatives in each V_{np}. In our simulation of the rectangular DR, we choose two types of basis functions: an entire-domain basis function which is an approximate solution for the fundamental mode of the DR [2], and a set of tetrahedral basis functions [5].

The entire-domain basis function is defined on the whole rectangular box, [2]

$$\vec{f}_e(x,y,z)=\left\{\hat{x}k_y\cos\left[k_x\left(x-\frac{a}{2}\right)\right]\sin\left[k_y\left(y-\frac{b}{2}\right)\right]\right.$$
$$\left.-\hat{y}k_x\sin\left[k_x\left(x-\frac{a}{2}\right)\right]\cos\left[k_y\left(y-\frac{b}{2}\right)\right]\right\}\sin(k_z z+\varphi) \quad (9)$$

where k_x, k_y and k_z are the solutions of the equations

$$\frac{k_x}{k_d^2-k_z^2}\tan\left(\frac{k_x a}{2}\right)=\frac{1}{k_0^2-k_z^2}\alpha_x \quad (10)$$

$$\frac{k_y}{k_d^2-k_z^2}\tan\left(\frac{k_y b}{2}\right)=\frac{1}{k_0^2-k_z^2}\alpha_y \quad (11)$$

$$\cot^{-1}\left(-\frac{\alpha_z}{k_{zd}}\right)-\cot^{-1}\left[\frac{k_{zs}}{k_{zd}}\cot(k_{zs}h)\right]=k_z d \quad (12)$$

$$k_{zs}\cot(k_{zs}h)=k_{zd}\cot\varphi \quad (13)$$

where

$$k_x^2+k_y^2+k_z^2=\varepsilon_r k_0^2 \quad (14)$$

$$k_x^2+k_y^2+k_{zs}^2=\varepsilon_{rs}^2 k_0^2 \quad (15)$$

$$k_x^2+k_y^2-\alpha_z^2=k_0^2 \quad (16)$$

$$-\alpha_x^2+k_y^2+k_z^2=k_0^2 \quad (17)$$

$$k_x^2-\alpha_y^2+k_z^2=k_0^2 \quad (18)$$

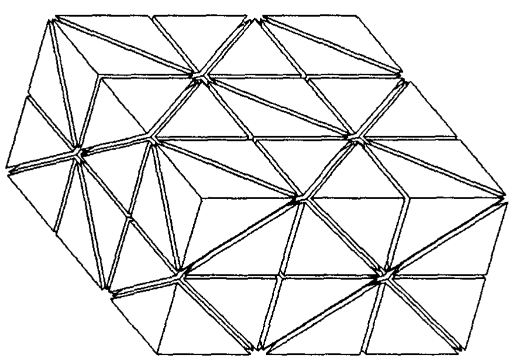


Fig. 2. A $3\times3\times2$ tetrahedral mesh for a rectangular DR.

The rectangular box is meshed into tetrahedrons, as shown in Fig.2. Each tetrahedral basis function is defined on either one tetrahedron or a pair of adjacent tetrahedrons [5]. The advantage of using tetrahedral basis functions is that the normal components of the basis function is none-zero only on one triangle of each tetrahedron, and is continuous on the common triangle of the pair of tetrahedrons Therefore many surface integrals in (8) can be omitted.

For (8) to have non-zero solution, the determinant of $[Z]$, as a function of frequency, must be zero. Therefore the complex resonant frequency of the DR is the solution of the following equation

$$|Z|=0 \quad (19)$$

III. Numerical Results

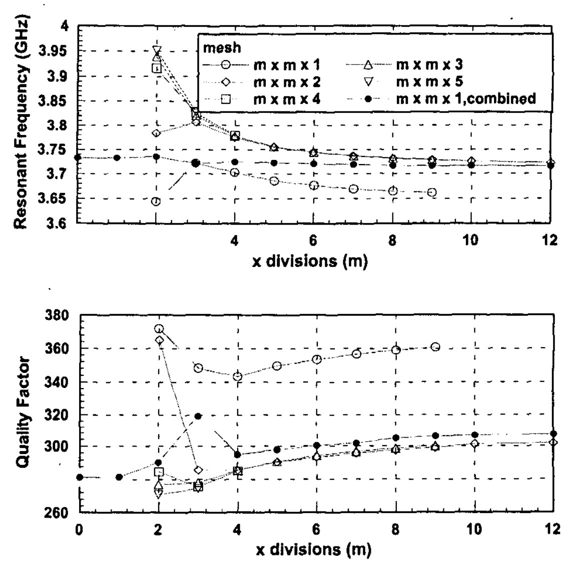
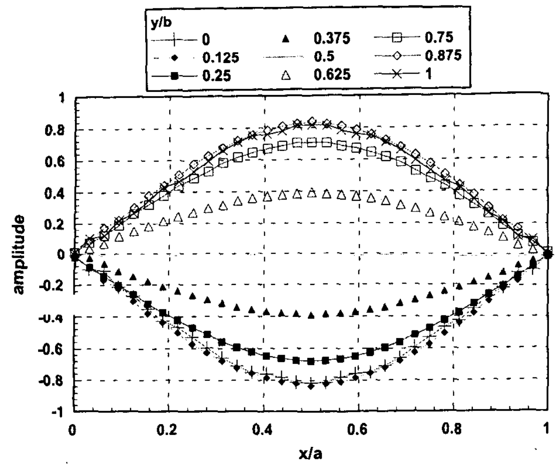


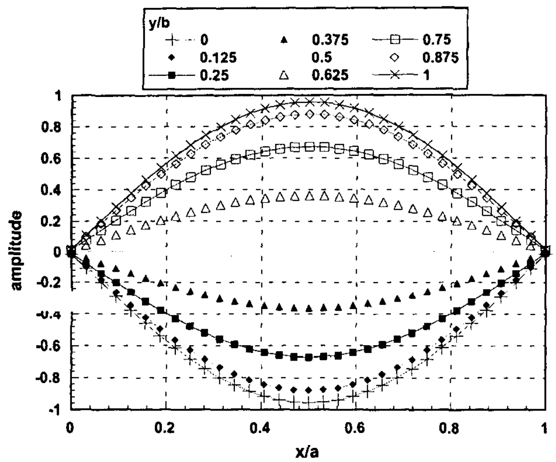
Fig.3. The resonant frequency and quality factor of a rectangular DR in MIC environment.

To verify this method and demonstrate its efficiency, the fundamental mode ($TE_{11\delta}^{z}$ mode) of a rectangular DR in MIC environment is simulated. The size of the DR is $a=b=14.98mm$ and $d=7.48mm$. The thickness of the substrate is $s=0.7mm$. The permittivity of the DR and the substrate is $\varepsilon_d=34.19\varepsilon_0$ and $\varepsilon_s=9.6\varepsilon_0$, respectively.

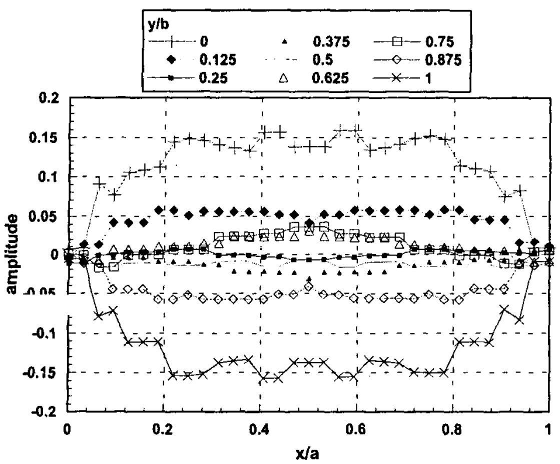
Fig.3 shows the convergence of this method and compared with those using only tetrahedral basis functions. Different meshes are used. Here a $m\times n\times p$ mesh means the number of divisions in *x*, *y* and *z* directions are *m*, *n* and *p*, respectively (Fig.2).



(a) total field



(b) contribution of entire-domain basis functions



(c) contribution of sub-domain basis functions

Fig.4 Modal field component E_x at plane $z = d/2$.

Fig.4 shows the E_x component on the plane $z = d/2$. Fig.4(a) is summation of Fig.4(b) and Fig.4(c). Since Fig.4(b) is already a good approximation of the exact field, only small refinements are needed to achieve better approximation. As shown in Fig.4(c), these refinements occur mainly near the y boundaries, and with opposite sign to the contribution of the entire-domain basis function. The amplitudes of these refinements are less than 15% of the contribution of the entire-domain basis function, and they are quite smooth therefore can be easily represented by sub-domain basis functions.

IV. CONCLUSIONS

In this paper a MoM-MPIE formulation using combined entire-domain and sub-domain basis functions is proposed and applied to the simulation of the fundamental mode of a rectangular DR in MIC environment. Numerical results show that this formulation is more efficient than the formulation using only sub-domain basis functions.

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